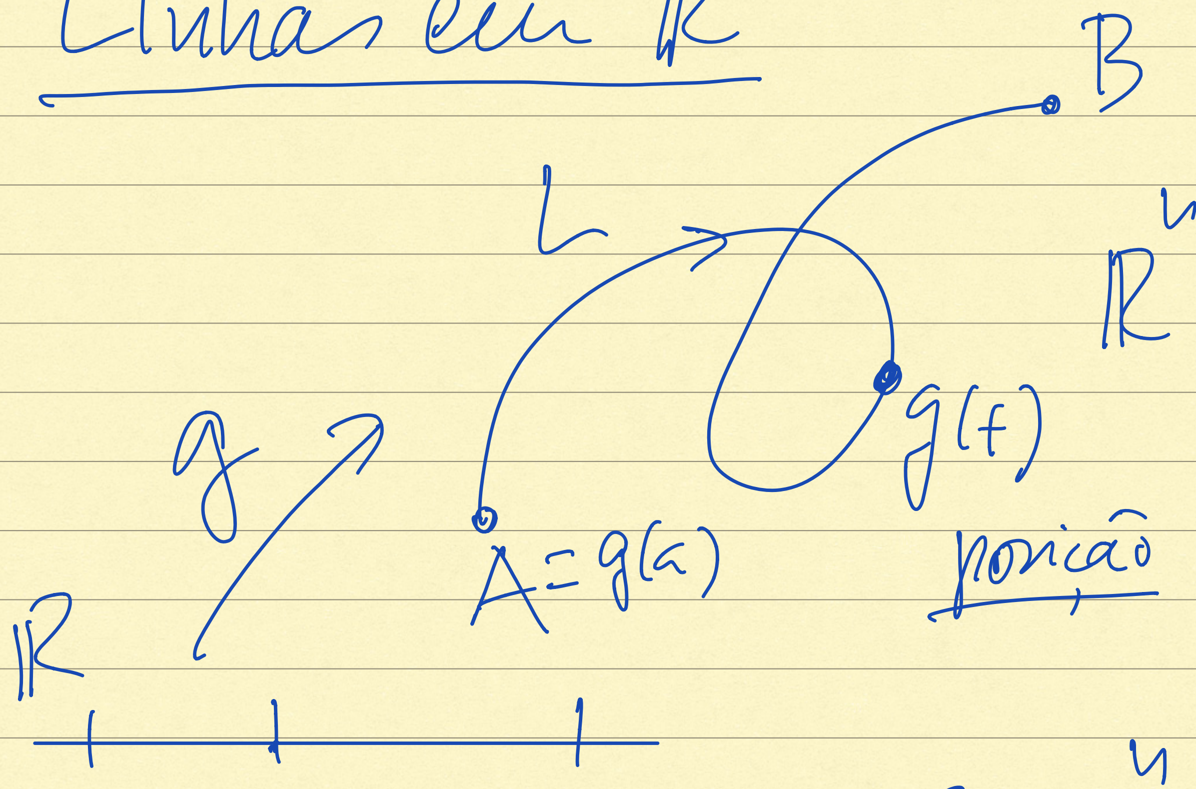
Linhas em $\mathbb{R}^n$



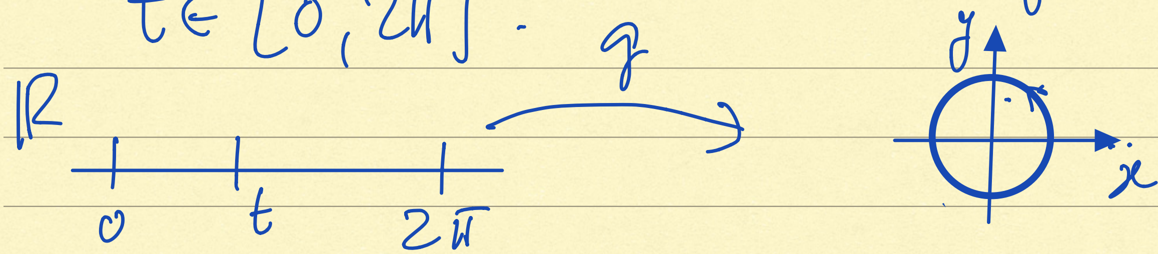
$a \qquad t \qquad b$

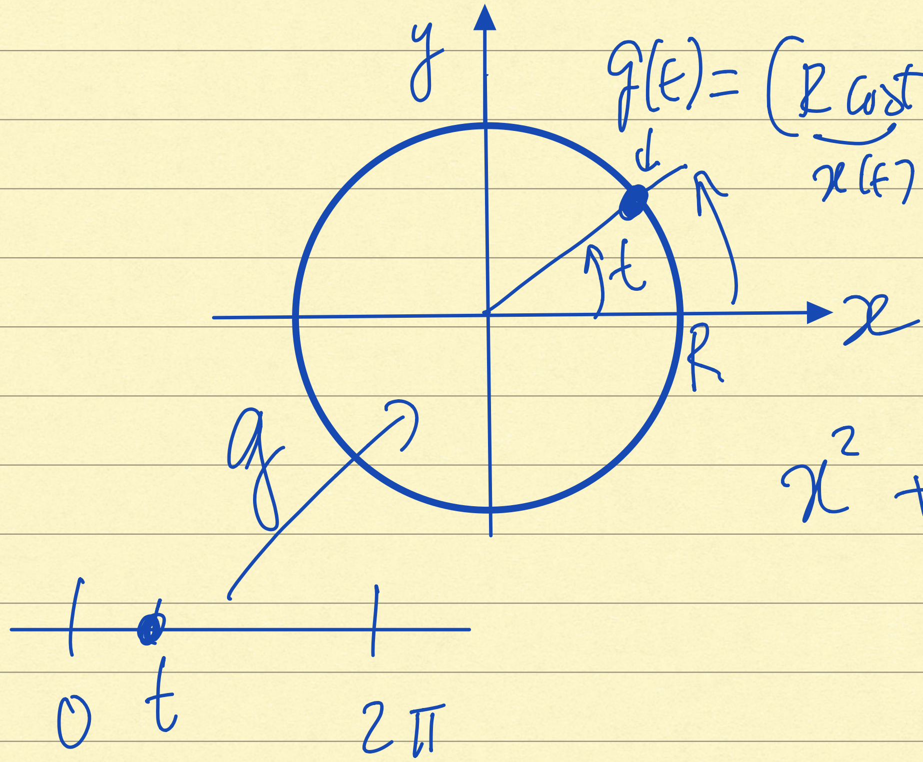
(instante)

$g : \mathbb{R} \to \mathbb{R}^n$ contínua

Exemplo 1: $g(t) = (\underbrace{R\cos t}_{x(t)}, \underbrace{R\,\text{sen}\, t}_{y(t)})$

$t \in [0, 2\pi]$.

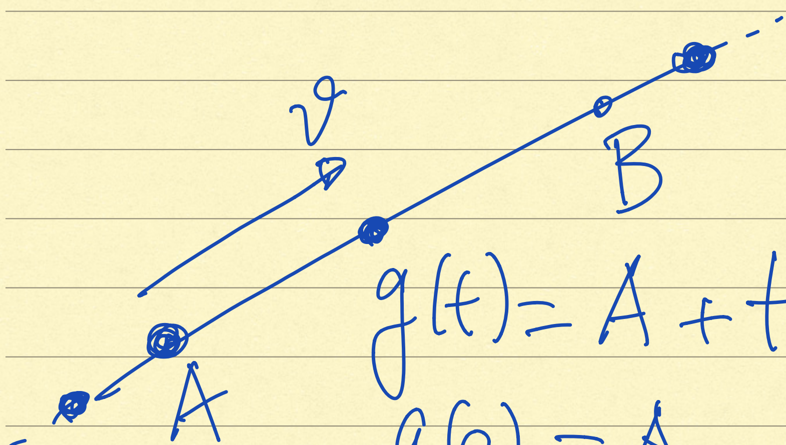




$$g(t) = (\underbrace{R\cos t}_{x(t)}, \underbrace{R\,\mathrm{sen}\,t}_{y(t)})$$

$$x^2 + y^2 = R^2$$

Exemplo 2: Linha reta



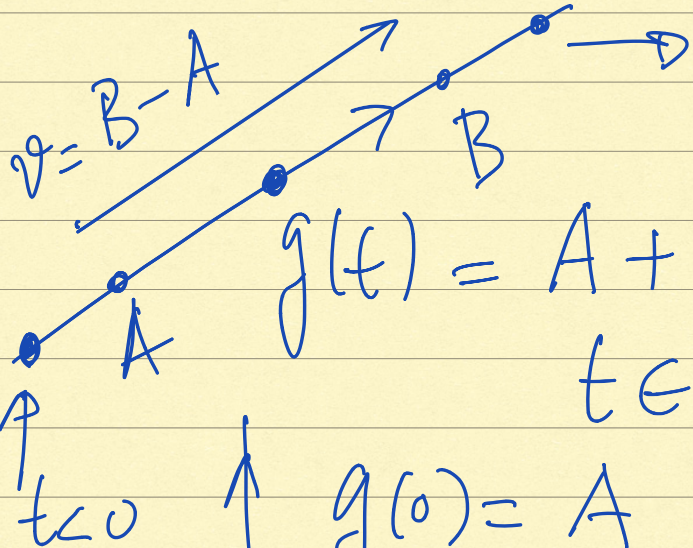
Dados A e v:

$$g(t) = A + tv, \quad t \in \mathbb{R}$$

$$g(0) = A$$

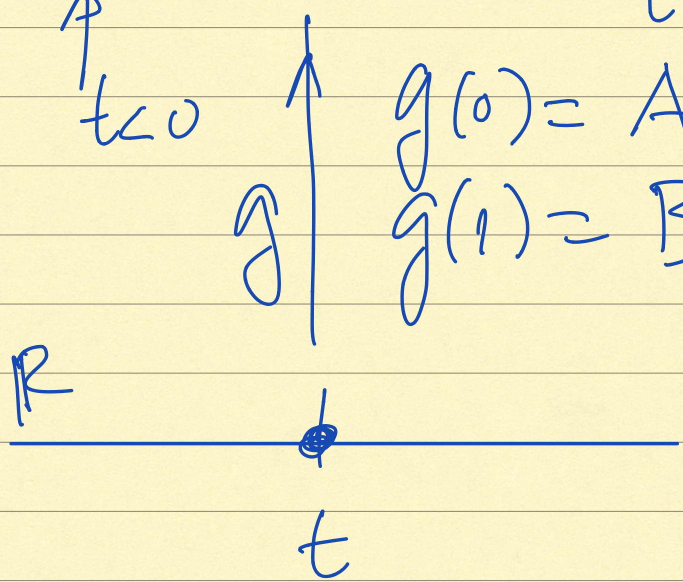
$$g(1) = A + v$$

Se tiver dados A e B:



$$g(t) = A + t(B - A)$$

$$t \in \mathbb{R}$$



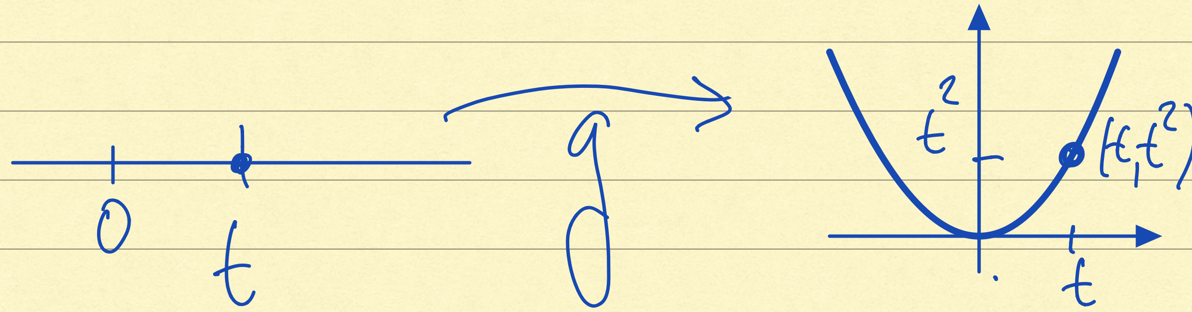
$$g(0) = A$$

$$g(1) = B$$

$g : \mathbb{R} \to \mathbb{R}^4$

<u>contínua</u>

<u>Exemplo-3</u>: Parábola $y = x^2$



$$g(t) = (t, t^2) = (x(t), y(t))$$

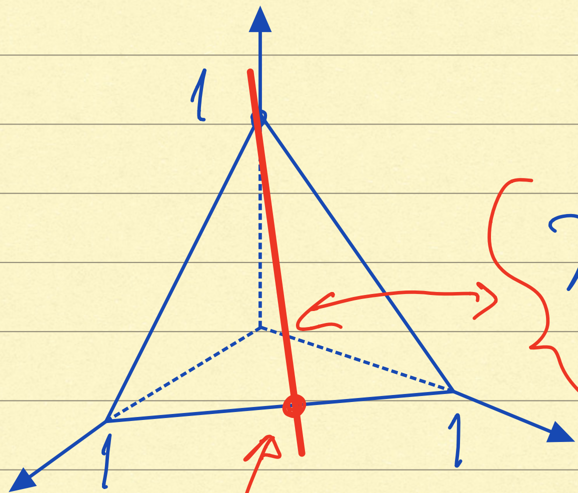
$$\boxed{y = x^2}$$

gráfico de $f(x) = x^2$

$f: \mathbb{R} \to \mathbb{R}$, contínua

$$\begin{cases} x + y + z = 1 \\ x = y \end{cases} \Leftrightarrow \begin{cases} z = 1 - 2x \\ y = x \end{cases}$$

$$(x, x, 1-2x) = g(x)$$

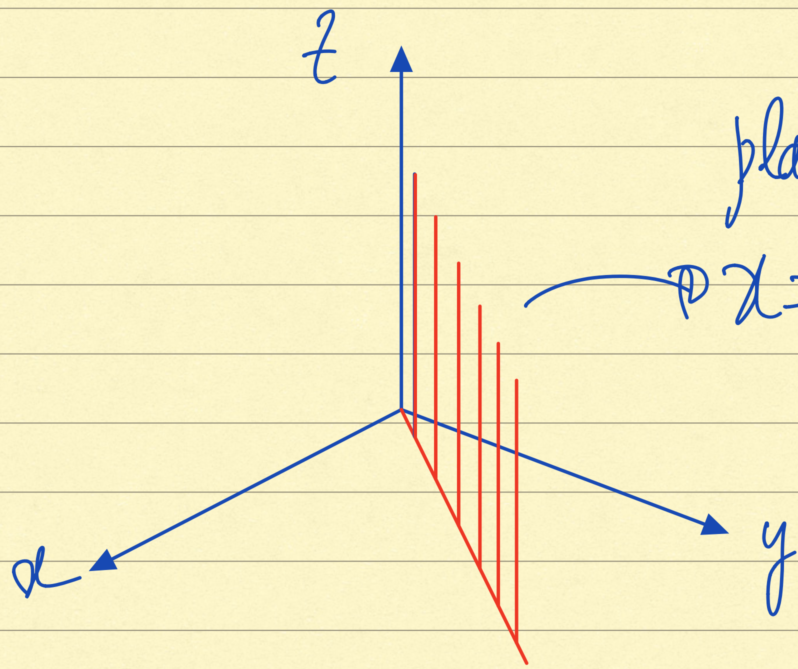


plano

$\left\{ \begin{array}{l} x+y+t=1 \\ x=y \text{ plano} \end{array} \right.$

recta

$\left(\frac{1}{2},\frac{1}{2},0\right)$



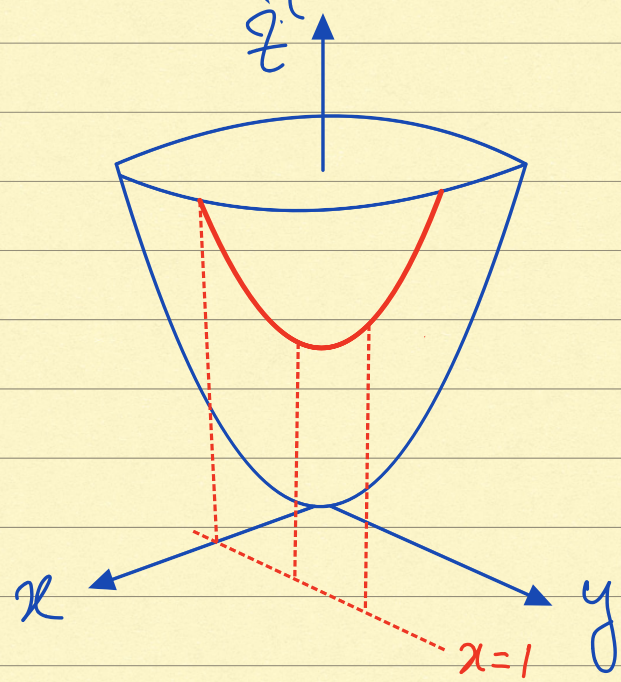
plano vertical

$\to x=y$

$$g(t)=(t,t,1-2t)$$

$t\in\mathbb{R}$

<u>Exemplo:</u> $\begin{cases} z = x^2 + y^2 \\ x = 1 \end{cases} \Leftrightarrow \begin{cases} z = 1 + y^2 \\ x = 1 \end{cases}$



$$g(t) = (1, t, 1+t^2)$$

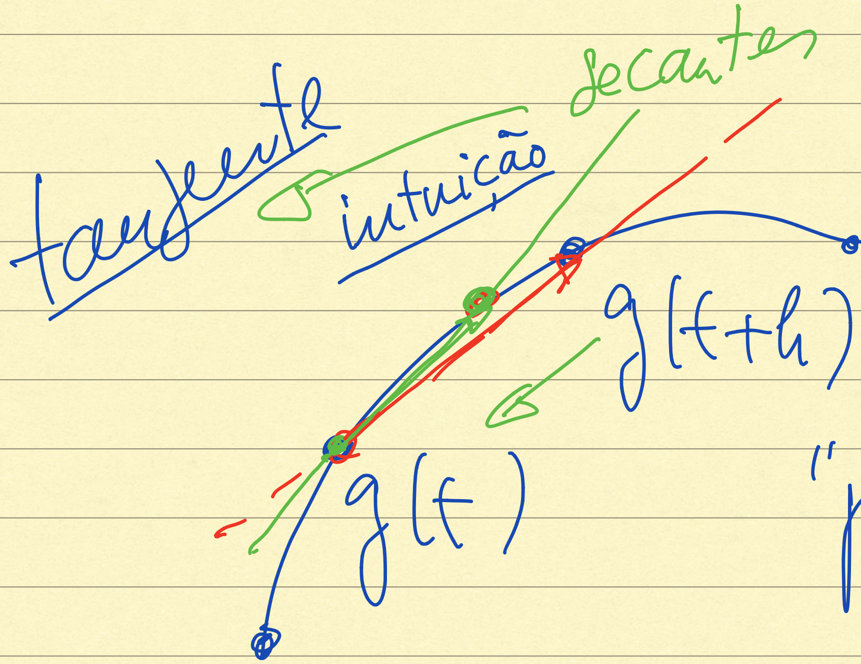
$g : \mathbb{R} \to \mathbb{R}^3$ contínua

$g: \mathbb{R} \to \mathbb{R}^n$, diferenciável

$t \mapsto g(t)$ 1 variável

$$g'(t) \equiv Dg(t) \quad n \times 1$$

$$= \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} \qquad \begin{bmatrix} \vdots \\ g'(t) \\ \vdots \end{bmatrix}_{n \times 1}$$



"parece" razoável

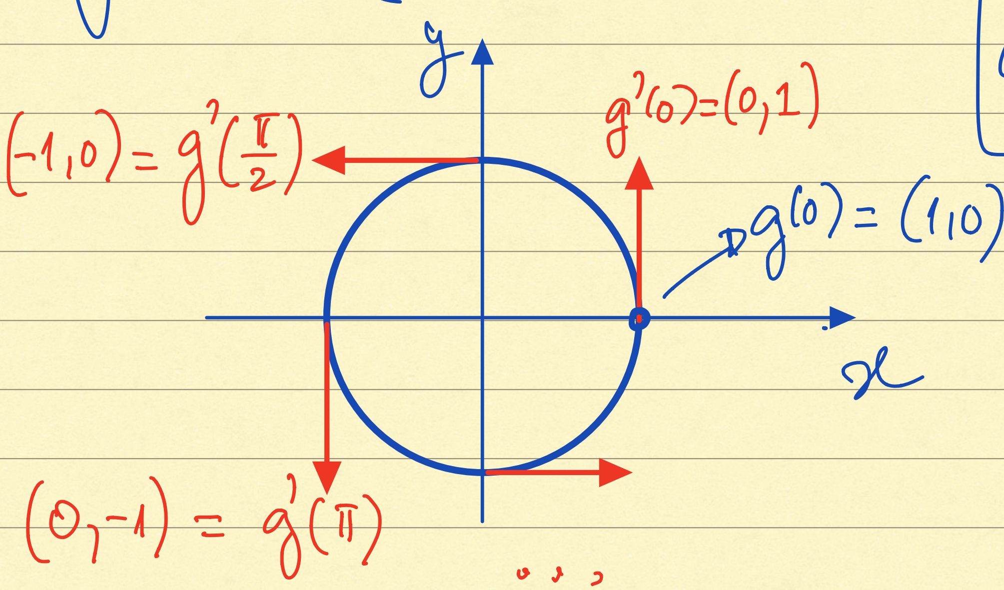
$\Rightarrow$ definição de tangente

Definição: $g'(t) = \lim\limits_{h \to 0} \dfrac{g(t+h) - g(t)}{h}$

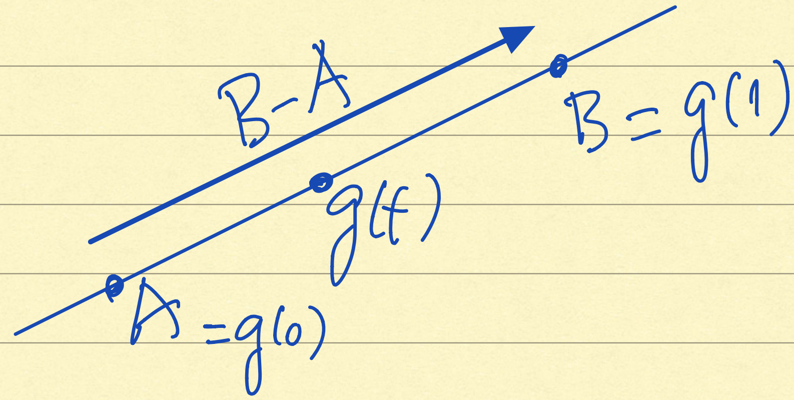
é o **vector tangente** à linha L no ponto $g(t)$.

Exemplo: $g(t) = (\cos t, \operatorname{sen} t)$

$$g'(t) = (-\operatorname{sen} t, \cos t) = \begin{bmatrix} -\operatorname{sen} t \\ \cos t \end{bmatrix}$$



$$g(t) = A + t(B-A), \quad t \in \mathbb{R}$$



$$g'(t) = B - A$$

"A recta é tangente a ela própria"